CURRENT DIVISION IN PLANE POSITIVE GRID TRIODE*

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ABSTRACT. The paper presents a simple electron optical treatment of the problem of current division in a plane triode by assuming that the openings between the consecutive grid wires behave as thin cylindrical lenses free from spherical aberration. Both the cases of reflection and no reflection of electrons in front of the anode are considered. It is shown that for the latter the results obtained from the expressions for current division given earlier by Spangenberg and by Jonker and Tellegen follow as special cases of the same general equation obtained here (under all practical conditions of operation). For the former, the expression obtained is shown to be identical with the one given earlier by Jonker. Attempt has also been made to examine the applicability of the relation obtained to the case of valves using filamentary cathodes. It is found that the experimental results for such valves, as reported earlier by Hamaker, generally agree well with those given by the theory.

The assumptions underlying the treatment are discussed critically and are shown to be quite reasonable for all practical purposes.

INTRODUCTION

The problem of the design of triodes as amplifier requires a knowledge of a number of tube constants, *viz.*, amplification factor (μ), transconductance (g_m) and dynamic plate resistance (r_p). For the problem of the design of high power amplifiers and oscillators it is also essential to know how the total current is divided between the grid and the anode. This division of the current has been found experimentally to depend on the tube geometry and on the ratio of the plate potential V_a and the grid potential V_g (Knoll and Schloemilch, 1934). Several attempts have been made to derive a theoretical expression for the ratio of the grid current I_g to the total current I_c in terms of the tube structure and electrode potentials. The methods of treatment are, however, difficult, involving a detailed consideration of the motions of the electrons within the inter-electrode space. Further, expressions thus derived are not in a form suitable for numerical calculations. It is the purpose of the present paper to show that quite satisfactory expressions for the current ratio can be obtained from simple electron optical considerations. Some of these expressions are also very convenient for numerical calculations.

2. EARLIER WORKS

For purposes of reference and comparison of results we first recall briefly the expressions for the current division, as obtained by earlier workers on the problem.

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Spangenberg (1940) considered the whole electronic path between the cathode and the anode without taking into account any possible strong variation of the field in the neighbourhood of the grid and obtained (for the case of no reflection of electrons in front of the cathode) the expression,

$$\frac{I_g}{I_c}=\frac{2r_g}{c}\left[1+\frac{V_g-V_{eg}}{2V_{eg}}\cdot\frac{1+\ln\left(\frac{4a}{r_g}\right)}{\ln\left(\frac{c}{2\pi r_g}\right)}\right] \tag{1}$$

where,

r_g=the radius of the grid wire,

c=the grid pitch,

a=the distance between the grid and the cathode.

V_{eg}=the equivalent grid plane potential (*vide* Sec. 3).

The effects of space charge, initial velocities of electrons and secondary emission are neglected.

Jonker and Tellegen (1945), following an earlier attempt by the latter, showed that a more satisfactory expression for the above case may be obtained by taking into account the possible strong deviation of electronic paths in the immediate vicinity of the grid wires. These authors carried out a detailed calculation of the extra moment gained by an electron in the above region and showed that,

$$\frac{I_g}{I_c}=\frac{2r_g}{c}\sqrt{\frac{V_g}{V_{eg}}}\left[1+\frac{V_g-V_{eg}}{2\,V_{eg}\,\ln\left(\frac{c}{2\pi r_g}\right)}\right]. \tag{2}$$

Jonker (1945) also considered the case when some of the electrons suffer reflection in front of the anode and are ultimately collected by the grid. The expression for this case was,

$$\frac{I_g}{I_c}=1-\frac{2}{\pi}\,\frac{\sqrt{V_{eg}\,V_a}}{V_g-V_{eg}}\,\ln\left(\frac{c}{2\pi r_g}\right), \tag{3}$$

the effects of space charge, initial velocities and secondary emission being neglected.

Eqns. (1), (2) and (3) are the commonly used expressions at present. We now proceed to derive the new expressions by electron optical method.

3. ELECTRON OPTICAL TREATMENT OF THE PROBLEM

To treat the problem theoretically by the electron optical method we start with the following assumptions:

(*i*) Lateral deviations of the electrons from their paths perpendicular to the cathode plane are entirely due to electron optical effects of the grid.

(*ii*) The openings between two consecutive grid wires behave as thin cylindrical lenses. The focal length f of such a lens, according to Davisson and Calbicks (1931, see also, Bull, 1945), is given by,

$$f = \frac{2V_{eg}}{E_2 - E_1}, \quad .. \quad (4)$$

where,

V_{eg}=the equivalent grid potential

$$= \frac{V_g + V_a/\mu}{1+\frac{1}{\mu}+\frac{1}{\mu}\frac{b}{a}-\frac{(\mu+2)c}{2\pi\mu a}\ln\cosh\left(\frac{2\pi r_g}{c}\right)}, \quad .. \quad (5)$$

b=the distance between the grid and the anode,

E_2=the gradient of electrostatic potential on the anode side of the grid, and,

E_1=the gradient of electrostatic potential on the cathode side of the grid.

(*iii*) The lens action is confined to a very small region of width nr_g about the plane passing through the centre of the grid wires—n being a numerical factor.

(*iv*) The image due to the lens is free from spherical aberration. Thus, for a parallel beam of electrons incident along the lens-axis, the deviation of a ray striking the lens at a point distant y from the lens-centre is given by

$$\theta = \frac{y}{f}. \quad .. \quad (6)$$

(*v*) The effects due to initial velocities of electrons, space charge and secondary emission are negligible.

The validities of some of the above assumptions will be discussed in Sec. 6.

We can put the expression for f in Eqn. (4) in a more practical form by noting that

$$E_2 = \frac{V_a - V_{eg}}{b}, \quad E_1 = \frac{V_{eg}}{a}.$$

Further, the term with $\ln\cosh\frac{2\pi r_g}{c}$ in the expression for V_{eg} in Eqn. (5) is generally very small compared to the other terms in the denominator. Hence we have,

$$V_{eg} \triangleq \frac{V_g + V_a/\mu}{1+\frac{1}{\mu}+\frac{1}{\mu}\frac{b}{a}}. \quad .. \quad (7)$$

Hence, from Eqn. (4),

$$= \frac{2\,(V_g+V_a/\mu)}{\dfrac{V_a\left(1+\dfrac{1}{\mu}+\dfrac{1}{\mu}\,\dfrac{b}{a}\right)-(V_g+V_a/\mu)}{b}-\dfrac{V_g+V_a/\mu}{a}}$$

$$= -\frac{2\,ab\,(V_g+V_a/\mu)}{a\,(V_g-V_a)+bV_g}$$

$$= -\frac{2\,b\,(1+\phi/\mu)}{(1-\phi)+b/a} \tag{8}$$

where,

$$\phi = \frac{V_a}{V_g}. \tag{9}$$

We are now in a position to investigate the problem of current division in a plane positive grid triode with a plane equipotential cathode, a plane equipotential anode and a plane equipotential grid composed of straight, equally spaced wires of identical diameters. We shall consider two cases:

(*i*) when there is no reflection of electrons in front of the anode, and

(*ii*) when there is reflection of electrons in front of the anode.

Case I: No reflection in front of the anode.—If $V_a > V_{eg}$ all the electrons emerging through the grid spaces are collected by the anode, there being no

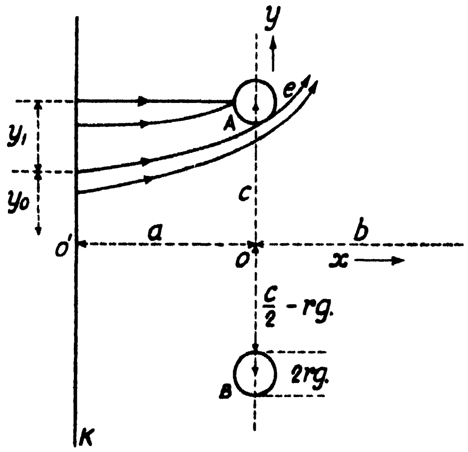

Fig. 1.

The nature of the trajectories of electrons in a plane positive grid triode when there is is no reflection.

reflection of electrons in front of the same, and the grid current is composed only of the forward moving electrons. Consider now figure 1, where K is a section of the cathode and A and B are sections of the grid wires (running perpendicular to the plane of the paper). The line OX, perpendicular to c and passing through the midpoint of AB is the axis of the electrostatic lens. An electron starting from O' on the cathode will not be deflected by the lens action. Those on either side will, however, be deflected—the deflection increasing from the axis O' outwards. Of particular interest is the electron which starts from a point on the cathode at a distance y_0 from O' such that it just grazes the grid wire surface A. All the electrons starting from points beyond y_0, that is, from points within the length $y_1 \left(= \frac{c}{2} - y_0 \right)$ will be captured by the grid wire A. And, those starting from within the length y_0 will reach the anode.

Under such condition,

$$\frac{I_a}{I_g} = \frac{y_0}{y_1} = \frac{y_1 + y_0}{y_1} - 1 = \frac{c}{2y_1} - 1, \qquad \ldots (10)$$

I_a being the anode current.

And, since

$$I_c = I_a + I_g,$$

$$\frac{I_g}{I_c} = \frac{2y_1}{c}. \qquad \ldots (11)$$

The value of y_1 may be obtained as follows: If v_y be the lateral velocity of an electron on emergence from the grid and v_x its longitudinal velocity, then,

$$\theta = \frac{v_y}{v_x} = -\frac{y}{f}. \qquad \ldots (12)$$

It may be noted that v_x remains practically unaffected by lens action, and hence the time during which the electron is subjected to lens action is $\frac{n\, r_g}{v_x}$, the numerical constant n taking account of the fact that the lens action is not confined to an ideally thin plane running through the centres of the grid wires, but extends a little to either sides of the plane. The average lateral velocity of the electron during the same time interval is $\frac{v_y}{2}$. Thus, the deviation of the electron from the path normal to the cathode plane is given by,

$$dy = -\frac{v_y}{2} \frac{n\, r_g}{v_x}$$

$$= -\frac{y}{2f} n\, r_g. \qquad \ldots (13)$$

For the limiting electron the lateral displacement is $y_1 - r_g = \frac{c}{2} - y_0 - r_g$.

Thus, we obtain after writing y_0 for y,

$$dy = y_1 - r_g = -\frac{y_0}{2f} n\, r_g = \frac{c}{2} - y_0 - r_g.$$

This gives,

$$y_1 = \frac{\frac{c}{2}\, n\, r_g - 2\, r_g f}{n\, r_g - 2f}. \qquad \text{.. (14)}$$

Substituting (14) in (10) one obtains,

$$\frac{I_a}{I_g} = \frac{c}{2y_1} - 1 = \frac{c}{2r_g} \frac{1 - \frac{2r_g}{c}}{1 - \frac{cn}{4f}} \qquad (15)$$

and substituting (14) in (11) one obtains,

$$\frac{I_g}{I_c} = \frac{1 - \frac{cn}{4f}}{\frac{c}{2r_g} - \frac{c}{4f}}. \qquad (16)$$

The value of n is not known; but, as the lens action can not extend beyond a region more that a few times the radius of the grid wire, we assume two values of n, namely 4 and 2. For $n=4$, we have

$$\frac{I_g}{I_c} = \frac{1 - \frac{c}{f}}{\frac{c}{2r_g} - \frac{c}{f}}. \qquad \text{.. (17)}$$

For $n = 2$

$$\frac{I_g}{I_c} = \frac{1 - \frac{c}{2f}}{\frac{c}{2r_g} - \frac{c}{2f}}. \qquad \text{.. (18)}$$

A useful parameter of a triode valve is the current division factor δ defined as the ratio of plate current to grid current for the special case of $V_a - V_g$ *i.e.*, $\phi = 1$.

Then, for $n = 4$ we have,

$$\delta = \frac{c}{2r_g} \frac{1 - \frac{2r_g}{c}}{1 - \frac{c}{f}} \qquad \text{.. (19)}$$

$$= \frac{c}{2r_g} \frac{1 - \frac{2r_g}{c}}{1 + \frac{c\mu}{2a(1+\mu)}}, \qquad \text{.. (20)}$$

and, for $n=2$,

$$\delta = \frac{c}{2r_g} \frac{1 - \frac{2r_g}{c}}{1 - \frac{c}{2f}} \qquad \text{.. (21)}$$

$$= \frac{c}{2r_g} \frac{1 - \frac{2r_g}{c}}{1 + \frac{c\mu}{4a(1+\mu)}} \qquad \text{.. (22)}$$

Case II: Electrons reflected from the front of the anode. If $V_a < V_{eg}$ some of the electrons which suffer lateral deviation will fail to reach the anode and turn back towards the grid, *i.e.*, there will be reflection from the front of the anode. The grid current has now two components—one due to the forward moving electrons and the other due to those returning after reflection.

To investigate the above problem we adopt the same line of reasonings as that of Jonker (1945). We first note that an electron emerging through the grid opening into the grid anode space has a total velocity of magnitude v_x given by

$$v_x = \sqrt{\frac{2e}{m} V_{eg}}. \qquad (23)$$

While the magnitude of this velocity changes with the motion of the electron, that of its component directed parallel to the anode remains constant. If $V_a < V_{eg}$

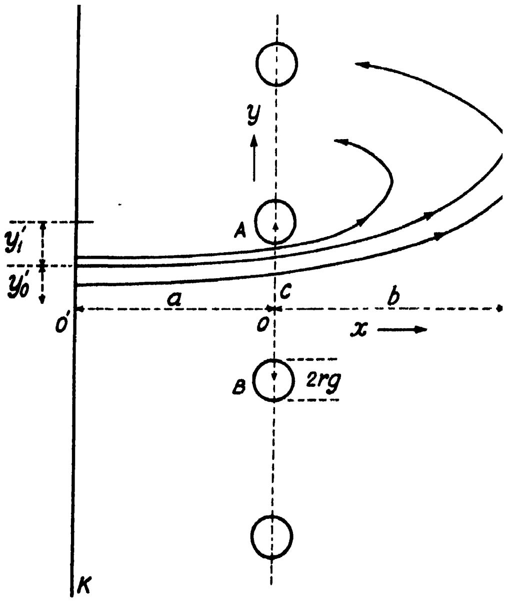

Fig. 2.

The nature of the trajectories of electrons in a plane positive grid triode when there is reflection.

the path within the grid-anode space will obviously be parabolic. The smaller the value of v_y the nearer will be the vertex of the parabola to the anode. The limiting case occurs when the value of v_y is such that the vertex of the parabola touches the anode surface. Let us suppose that this occurs for an electron striking the lens at a point distant y_0 from the lens centre (figure 2). Then, if we take

$$y_1' = \frac{c}{2} - y_0', \quad \ldots \quad (24)$$

all electrons originating from a region $2y_1'$ on the cathode, opposite the grid wires, will be collected by the grid, either, directly or after reflection. Thus, in this case,

$$\frac{I_g}{I_c} = \frac{2y'_1}{c}. \quad \ldots \quad (25)$$

If θ_1 be the angular deflection of the limiting electrons due to the lens action, then the part of the kinetic energy due to the component of the motion towards the anode is,

$$\tfrac{1}{2}\, m\, v_x^2 \cos^2 \theta_1.$$

Also, since the electron just fails to reach the anode, this energy must be used up entirely in gaining the potential energy $e\ (V_{eg} - V_a)$ within the grid-anode space. Thus, we obtain after Jonker,

$$\theta = \sqrt{\frac{V_a}{V_{eg}}}$$

We now make use of Eqns. (12) and (8) to obtain the value of y_1' thus

$$\frac{c}{2} - y_0' = \frac{c}{2} + f\sqrt{\frac{V_a}{V_{eg}}} \quad \ldots \quad (26)$$

Hence, from Eqn. (25),

$$\frac{I_g}{I_c} = 1 + \frac{2f}{c}\sqrt{\frac{V_a}{V_{eg}}}. \quad \ldots \quad (27)$$

This is expressible in the form

$$\frac{I_g}{I_c} = 1 - \frac{4ab\,(1+\phi/\mu)}{c[a\,(1-\phi)+b]}\sqrt{\frac{\phi\left(1+\frac{1}{\mu}+\frac{1}{\mu}\frac{b}{a}\right)}{1+\phi/\mu}}. \quad \ldots \quad (28)$$

4. COMPARISON WITH OTHER EXPRESSIONS

The usefulness of Eqns. (17), (18), (20), and (22) can best be judged by comparing the results obtained from them with those obtained from Spangenberg's formula [Eqn. (1)] and from Jonker and Tellegen's formula [Eqn. (2)]. As an illustrative example we take the tube geometry as considered by Jonker and Tellegen, *viz.*,

$$a = b = c,$$

and $\frac{2r_g}{c} = 0.064.$

In figure 3(*a*) values of $\frac{c}{2r_g}\frac{I_g}{I_c}$ are plotted against ϕ, for this tube geometry, as obtained from Eqn. (1). The points obtained from Eqn. (17) are shown by cross marks. These points fit remarkably well in the curve obtained from Eqn. (1). It thus appears that Eqn. (16) with $n=4$ gives the same results

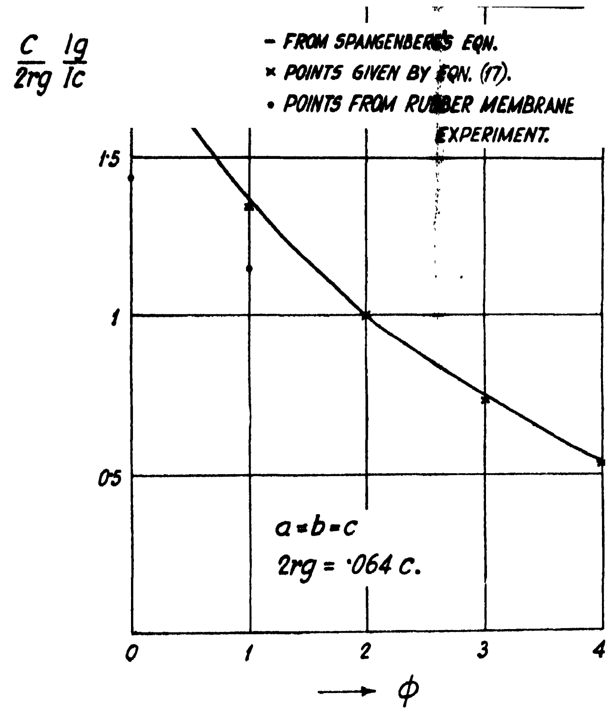

Fig. 3(*a*)

Comparison of results obtained from eqns. (1) and (17)

as those given by Eqn. (1). This conclusion is further substantiated by the results recorded in Table I where values of δ for several valve geometries calculated from Spangenberg's relation and from Eqn. (20) are compared. The agreement between the two sets of values is surprisingly good.

TABLE I

$\frac{a}{r_g}$	$\frac{c}{r_g}$	μ	δ From Spangenberg's relation Eqn. (1)	δ From Eqn. (20)
20	20	7·5	6·2	6·2
25	20	7·5	6·5	6·5
15	20	7·5	5·6	5·6
20	22	4	7	6·9
46	20	7·5	7·4	7·5
30	20	20	6·7	6·8

In figure 3 (b) values of $\frac{c}{2r_g}\frac{I_g}{I_c}$ are plotted against ϕ, for the above tube geometry, as obtained from Eqn. (2). Points obtained for the same tube from Eqn. (18) are shown by cross marks, and those obtained from experiments with a rubber membrane model by Jonker and Tellegen (1945) are marked by circular

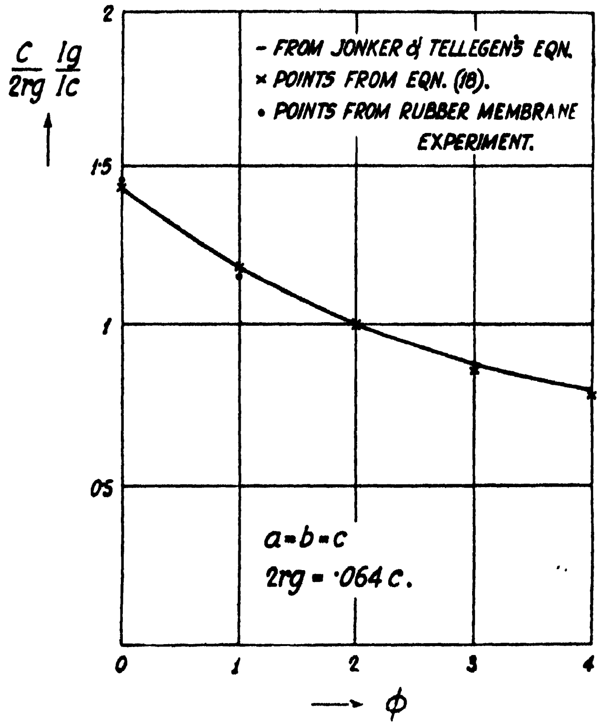

Fig. 3(b)

Comparison of results obtained from eqns. (2) and (18)

dots. It is seen that points given by Eqn. (18) lie closely on the curve and that Eqn. (16) with $n = 2$ gives, almost the same results as those given by Eqn. (2). Further, results obtained from experiments with rubber model agree more favourably with the values obtained with $n = 2$. Thus in actual tube lateral deviation appears to occur sharply within a very small region.

It should, however, be noted that results obtained from Eqn. (2) are, in general, somewhat different from those obtained with Eqn. (18) when V_g is small compared to V_a. Thus, while Eqn. (2) requires that I_g should become zero when $V_g = 0$, Eqn. (18) shows that this may be so even when V_g has a small positive value. To be precise, Eqn. (18) requires that

$$\text{for } I_g = 0, \ f = \frac{c}{2};$$

i.e., when the focal length of the grid aperture becomes equal to half its width. Under this condition y_1 in Eqn. (11) becomes zero. If V_g is reduced further, the value of c is effectively diminished in conformity with the relation $f=\frac{c}{2}$ and hence the grid current remains nil. Only in case of valves having special geometry, conditions for $I_g=0$ as required by Eqns. (2) and (18) become identical.

It will be noted from Eqns. (8) and (10) that the focal length of the grid slot becomes infinity when ϕ has the value,

$$\phi=1+\frac{b}{a},\ i.e.,\ \frac{V_a}{V_g}=1+\frac{\text{grid-anode distance}}{\text{grid-cathode distance}}.$$

Under this condition,

$$\frac{I_g}{I_c}=\frac{2r_g}{c}=\text{the screening constant of the valve.}$$

For smaller values of ϕ the lens behaviour is diverging and

$$\frac{I_g}{I_c}>\frac{2r_g}{c}.$$

For larger values of ϕ the lens behaviour is converging and,

$$\frac{I_g}{I_c}<\frac{2r_g}{c}.$$

For the tube considered above the critical value of ϕ is 2 and is greater or less than the screening constant according as ϕ is lesser or greater than 2. These facts are clearly understood from an inspection of figures 3 (a) and 3 (b).

It should be mentioned that the boundaries of the types of electrostatic lens, as constituted by the potential fields in the vicinity of two grid wires, are never well defined. In most cases the actual lens thickness may exceed the geometrical width $2r_g$ of the grid openings. A thickness of $4r_g$ as assumed in deduction of Eqns. (17) and (20) is, however, an over estimate for ordinary conditions of valve operation. Such large values may be approached only for low μ valves when the electrostatic fields on the two sides of the grid are such that equipotential surfaces on one side penetrate the openings in the grid and bulge out appreciably on the other side. Therefore, it may be concluded that Eqns. (2), (18) and (22) are more reliable for practical purposes.

The usefulness of Eqn. (28) may be best judged by comparing it with Eqn. (3). It may be shown that the relations expressed by the two equations are, in fact, identical. For this, one may borrow the following relation from Jonker and Tellegen (1945):

$$[E_1-E_2]\frac{c}{2\pi}\ \ln\frac{c}{2\pi r_g}=V_g-V_{eg},$$

$$\text{Or, } \frac{1}{E_2-E_1}=-\frac{c}{2\pi}\frac{1}{V_g-V_{eg}}\ln\left(\frac{c}{2\pi r_g}\right). \qquad (29)$$

From Eqns. (4), (26), (28) and (29) one easily obtains Eqn. (3). It may be noted that Eqn. (28) is true if $\phi << 1$. For the tube considered above this equation gives consistent results if $\phi < 0\cdot1$. For higher values of ϕ one must use Eqn. (18) or (22).

5. APPLICATION TO VALVES WITH FILAMENTARY CATHODES

In the present section we shall examine how far Eqn. (16), deduced for the ideal case of a plane, equipotential cathode, is applicable to filamentary cathodes of V or W shape as employed in practice.

It may appear at first sight that the ratios $\frac{I_g}{I_c}$ for triodes with such cathodes would depart widely from those given by Eqn. (16). However, Kosunose (1929) has shown that under the usual conditions of operation (*viz.* $V_g < V_a$) the plate current in such a triode is satisfactorily accounted for if it is assumed that the action of the filamentary cathode is equivalent to that of a plane equipotential strip of width twice the grid to cathode distance. Hence, it may be expected that Eqn. (16) would be applicable to such triodes, at least approximately, under the usual conditions of operation. To test this, we make use of the experimental results of Hamaker (1948) on the ratio of the grid current to the total current for four valve types with different valve parameters as shown in columns 1 and 2 of Table II. In column 4 of the same table are given the observed values of $\frac{I_g}{I_c}$ for different values of ϕ and in columns 5 and 6 the corresponding values as calculated from Eqn. (16) by putting $n=2$ and $n=4$ respectively.

A glance at the table shows that considering the very wide gulf between the idealised plane cathode and the actual filamentary cathode, the agreement in some of the cases is surprisingly good. Closer inspection shows that there is a regular trend in the closeness of the agreement. Thus, consider the figures in Table II enclosed between horizontal lines for each of the four types of valve.

It will be seen that for the case $\phi \geqslant 1$, the agreement between the observed and the calculated values of $\frac{I_g}{I_c}$ is better when ϕ is generally much larger than 1 and the value of n is taken as 2. For the rest of the cases agreement is better if n is taken as 4. The reason for this trend may be understood as follows: It will be recalled that for the plane cathode the field was uniform and its intensity on the cathode side of the grid was given by $\frac{V_{eg}}{a}$. For the case of filamentary rathode, however, the field is non-uniform and an expression like $\frac{V_{eg}}{a}$ gives only the average value of the field. From the geometry of the filament and the grid it is evident that the field near the cathode is more intense and that near the grid less intense than the average value of the field $\frac{V_{eg}}{a}$. It, therefore, follows that if the focal length is calculated from Eqn. (4) using the average value of the field, the true value of f is less than the calculated value. Hence, if the intensity of the field near the grid due to the grid potential is not much affected by anode potential, *i.e.*, if V_a is not much larger than V_g, then the departures in the values of $\frac{I_g}{I_c}$ from the case of the plane cathode (due to the actual focal length being smaller) may be compensated by taking a larger value of n. On the other hand, if V_a is much larger than V_g, then the field intensity near the grid is determined more by the anode potential which tends to make the field on the cathode side of grid uniform. Hence, calculated values of f agree better with the actual values and no compensation by taking a larger value of n is necessary. These considerations explain in a qualitative manner the trend of agreement between the observed and the calculated values.

TABLE II

Valve type	Valve parameter	$\phi=\frac{V_a}{V_g}$	Values of I_g/I_c. Experimental	From Eqn (16) $n=2$.	From Eqn. (16) $n=4$.
(1)	(2)		(4)	(5)	(6)
KZ141	$a=2{\cdot}25$ mm $b=2{\cdot}5$ mm	0·5 1	·2 ·12	·11 ·1	·13 ·117
	$c=2{\cdot}1$ mm $2r_g=\ {\cdot}18$ mm $\mu=6$	2·5 5 10	·09 ·06 ·04	·085 ·058 ·034	·08 ·03 0
KZ139	$a=2{\cdot}25$ mm $b=2{\cdot}5$ mm $c=1{\cdot}0$ mm	·5 1 2·5	·28 ·22 ·17	·203 ·2 ·175	·225 ·21 ·171
	$2r_g=\ {\cdot}18$ mm $\mu=24$	5 10	·13 ·1	·143 ·09	·104 0
KZ104	$a=2{\cdot}25$ mm $b=2{\cdot}5$ mm $c=1{\cdot}4$ mm $2r_g=\ {\cdot}18$ mm	·5 1 2·5 5	·25 ·16 ·1 ·07	·15 ·14 ·118 ·1	·173 ·16 ·108 ·07
	$\mu=10$	10	·055	·06	0
TC1/75	$a=2{\cdot}0$ mm $b=8$ mm	·4 1	·219 ·15	·126 ·122	·15 ·14
	$c=1{\cdot}75$ mm $2r_g=\ {\cdot}18$ mm $\mu=24$	4 10 40	·105 ·081 ·033	·107 ·085 ·031	·112 ·066 0

It will be seen from Table II that the theoretical values of $\frac{I_g}{I_c}$ for $\phi < 1$ are always smaller than the experimental values. This lack of agreement may be due to several reasons. Firstly, for such values of ϕ, the grid potential plays a highly dominant rôle in controlling the valve action and the effect due to space charge becomes considerable on both sides of the grid. This makes the values of E_1 and E_2 uncertain—particularly that of the latter (Bull, 1945). Secondly, because of the excessive influence of grid potential the field distribution between the cathode and the grid may become so much different from that of a triode having plane equipotential cathode that Kosunose's approximation ceases to be applicable. Lastly, because of the predominant rôle of grid potential, the field E_1 on the cathode side may become so much smaller than the average value $\frac{V_{eg}}{a}$ that to compensate for this a value of n much larger than 4 has to be taken.

6. DISCUSSION ON THE ASSUMPTIONS MADE

We conclude the paper by checking how far the assumptions made in the above treatment are justified.

The assumption that the lateral deviation suffered by the electrons is due only to the electron optical action of the grid is approximately valid if, (*i*) the lateral component of the field in the cathode-grid space is a very small fraction of electron optical field and (*ii*) if the grid plane is a truly equipotential surface. Now, the lateral component of the field is given by (Spangenberg, 1948)

$$E'_y = -\frac{2\pi\, q_g \sin \pi\left(1-\frac{2y}{c}\right)}{c\left[\cosh\left(\frac{2\pi x}{c}\right) - \cos \pi\left(1-\frac{2y}{c}\right)\right]},$$

where q_g is the linear charge density of the grid wire given by

$$\frac{V_g\, c\,[\,a\,(1-\phi\,)+b\,]}{4\,ab\left[1+\frac{1}{\mu}+\frac{1}{\mu}\frac{b}{a}\right]} = -\frac{c\,V_{cg}}{2\pi f} = -\frac{c\,[\,E_2-E_1\,]}{4\pi},$$

x is the axial distance and y the lateral distance measured in the manner as shewn in figures 1 and 2. It is evident that the value of E'_y remains extremely small for large values of x and small values of y. Now, for the cathode grid space bounded on one side by the grid openings, the smallest value that x can have is $-r_g$ and the largest value that y can have is $\frac{c}{2}-r_g$. Thus, at the point $\left(-r_g, \frac{c}{2}-r_g\right)$ the value of E'_y will be a maximum and will be given by

$$E'_y = \frac{c\,[\,E_2-E_1\,]}{4\,\pi\, r_g}.$$

But, near this point the electron optical field is, to a first approximation, given by

$$E_y = y\,\frac{dE}{dx} = \left(\frac{c}{2} - r_g\right)\frac{E_2 - E_1}{2r_g}.$$

$$\therefore \quad \frac{E'_y}{E_y} \simeq \frac{1}{\pi} < 1,$$

$$\text{Or,} \quad E'_y < E_y.$$

Thus, even at the point where E'_y is strongest, electron optical field plays the dominant rôle.

The actual grid is, of course, not a truly equipotential surface; but, in making calculations use has been made of the equivalent grid potential which is based on the concept of a plane equipotential grid.

The assumption that the openings in the grid behave as thin lenses, not introducing spherical aberration, is true to a first approximation. This is seen as follows. Firstly, it has been found that satisfactory results are obtained if the lens action is assumed to extend over a thickness which is always much smaller than the focal length f and the aperture c. Secondly, although a single isolated cylindrical aperture gives rise to considerable spherical aberration, it is not so for the case of a set of such lenses placed side by side (as in a plane triode). This is because, the mutual influence of the adjacent apertures acts in such a manner as to reduce the effect of spherical aberration to a large extent. This has been verified by Jonker (1945) by experiments with rubber models. It has been found, in fact, that Eqn. (6) holds right up to the extremeties of the grid opening in a plane triode.

The assumptions that there are no effects due to (i) space charge and (ii) to the initial velocity are not strictly correct. However, one may take account of these effects as follows: (i) Tellegen (1926) has advanced the view point that since for ordinary valves the effect of space charge is confined mostly within the cathode-grid space, its effect on off-cathode field would only cause an apparent reduction of a, say by 25%. Thus the equations developed in Sec. 3 may be applied to a space charge limited valve by substituting $\frac{3}{4}a$ for a. (ii) The initial velocity components directed normal to the cathode surface have no doubt a Maxwellian distribution. Chaffe (1942) has, however, shown that in all cases where it plays significant rôle, the effect can be taken into account—to a reasonably good approximation—by adding a suitable small correlation V_m to V_a. Thus, the equations developed will hold for ordinary valves if V_a is replaced by $V_a + V_m$ However, it is easily seen that this correction does not alter the calculated values materially. It is to be noted that the electrons also possess a lateral velocity of the thermal agitational type. But, this does not affect the position, because, its effect will be merely to superpose a partition type of noise upon the steady electrode currents.

CONCLUDING REMARKS

The proposed electron method of treatment of the problem of current division in a plane triode is found to give quite satisfactory results.

It is to be pointed out that the equations developed here give the current ratio in terms of a single pair of dimensionless quantities, $\frac{2r_g}{g}$ and $\frac{f}{c}$ both of which may be obtained readily. This makes the numerical calculations very easy. Further, since $\frac{2r_g}{c}$ represents the screening constant and $\frac{f}{c}$ the relative aperture of the lens the equations may easily be generalised so as to include the case of valves of any geometry and for plane valves having any form of grid. (This will form the subject matter of a future communication).

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